Calculus Summer Packet
For students taking Calculus 2024-2025

- This packet will count as your first homework assignment and is due the first day of class.
- There will be a test on this material in the first two weeks of school.
- Answers are provided on the last page of the packet.
- You must do your work on separate pages and show all work to receive full credit.
- The purpose of this packet is to keep math topics fresh in your mind over the summer and also to assess your knowledge of topics essential to our work in Calculus.
- Do at least half of the problems (either evens or odds) and extra in any sections that you feel weaker in.

Summer Assignment Prior to the Start of Calculus Summer 2023 Name:_____

Do at least half of these problems (either evens or odds). If there is a section you feel like you need more practice on, do more problems from that part.
There will be a test on the material covered in this packet in the first few days of school. You are expected to be able to do all of the following problems **without** a calculator.
Skills you are expected to have mastered:

- Definitions of mathematical terms such as: term, factor, variable, constant.
- Ability to identify like terms (even terms with logs or exponentials in them).
- PEMDAS
- Exponent rules
- Factoring
- Fractions

1) Fill in the following table by listing all coefficients, like terms, and constants. In the fourth column write a count of the number of distinct terms (in other words, how many un-like terms are there):

	Coefficients	**Like Terms**	**Constants**	**Distinct Terms**
$3x + 7$				
$4m + (-3n) + n$				
$6kp + 9k + kp - 14$				
$-8y + 6ab + 7 - 3ba$				
$c + 2c - 5c + 1$				
$5x^2 - x^2y + 4yx^2 + xy^2$				
$\pi r^3 + 2r^2 - 2\pi + 8r^3$				
$3\mathrm{x} \cdot \log(x) + x - \ln(7)\, x$				
$xe^x + 2^x - 5xe^x - 8$				

2) Fill in the following table by listing all coefficients, like terms, and constants. In the fourth column write a count of the number of distinct terms (in other words, how many un-like terms are there):

	Coefficients	**Like Terms**	**Constants**	**Distinct Terms**
$5x + 7y - 7$				
$5a + 2b - 3a + 4$				
$4st - 5s + 3ts + 6$				
$7a^3b^4 + b^4a^3 + 4a^3b - 1$				
$\pi + x^2 + \pi y^2 - 2\pi x^2$				
$5\mathrm{x}^2 \cdot \ln(x) + 8x - \log(7)$				
$3^x + x8^x - e^x + 1$				

Explanation here: http://mathcentral.uregina.ca/QQ/database/QQ.09.07/h/maddie1.html

If you need review look through this material (search for khan academy expression terms factors coefficients): https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-evaluating-expressions/v/expression-terms-factors-and-coefficients

Also this:
https://www.mathsisfun.com/algebra/like-terms.html

3. Give an example of an expression:

4. Give an example of an equation:

5. Make up a term. Write it and circle the numerical coefficient.
coefficient's value?

6. Write a term with an "invisible" numerical coefficient. What is the

7. PEMDAS: Solve the following with your calculator. The answer is $x = 1.27$. If that's not what you got, figure out why.

$$8 = 2\pi x$$

8. PEMDAS: Solve

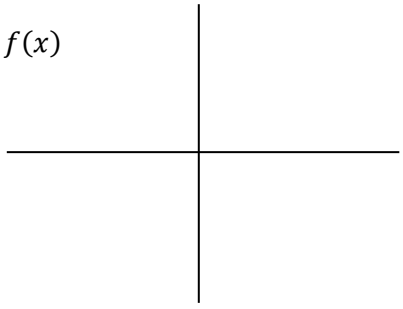
$$8 = \frac{(2x-4)^2}{2}$$

Not enough space is provided for the following questions. Work out your answers on a separate sheet of paper.

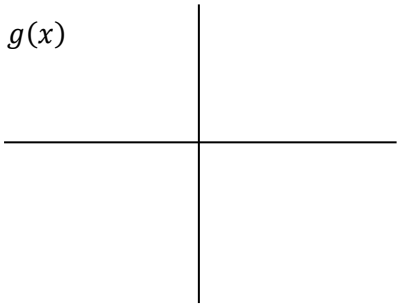
9. A fellow student sees the expression $\frac{x+6}{6}$ and thinks they will save themself trouble by cancelling the sixes to get $\frac{x+1}{1}$
Explain to your friend why they have made an error.

10. Your friend protests that the expressions are equal when they plug in zero for *x*.
Graph $f(x) = \frac{x+6}{6}$ and $g(x) = \frac{x+1}{1}$ Then refer to the graphs to explain to your friend why $f(0) = g(0)$ but in general $f(x)$ does not equal $g(x)$

$f(x)$



$g(x)$



11. Your friend is still struggling. Later on they see the following function and simplify it like so:

$f(x) = \frac{3x}{x-2}$

$= \frac{3}{1-2}$

$= -3$

1) Plug in specific x values and compare them to -3 to convince your friend that they cannot cancel this way.

2) Use a graph to convince your friend that cancelling this way is not valid.

12. Still later your friend sees you make the following cancellation:

$f(x) = \frac{3x}{x^2-2x}$

$= \frac{3}{x-2}$ Your friend is upset, nay! Betrayed! Can you convince them that your simplification is allowed? Or have you made a mistake?

Simplify each expression

You will be asked to add, multiply, divide, and simplify fractions. Here is a good site to review fractions: www.chilimath.com/lessons/advanced-algebra/simplifying-complex-fractions/

Dividing fractions tends to give students trouble. Review this topic here: www.mathsisfun.com/fractions_division.html

13) $\frac{-2}{3} - 5$

14) $\frac{1}{6} + \frac{8}{4}$

15) $\frac{5}{9} - \frac{3}{4}$

16) $\frac{1-\frac{1}{3}}{8}$

17) $\frac{12}{\frac{3}{4}+2}$

18) $\frac{\frac{4}{5}+\frac{1}{9}}{2}$

19) Solve for x. $\frac{3}{7} = \frac{x}{4}$

20) Solve for x. $\frac{9}{4} = \frac{2}{x}$

21) Solve for x. $x = 3 + \frac{28}{x}$

22) Solve for x. $x = \frac{1+\frac{1}{x}}{2}$

Simplify each expression. Don't use any decimal coefficients.

23) $\frac{x^2+7x+12}{x+3}$

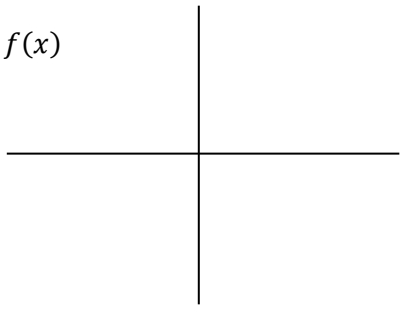
24) $\frac{x^2+6x+5}{x^2-1}$

25) A fellow student sees the expression $\sqrt{x^2+4}$ and thinks they see a nice simplification: $\sqrt{x^2} + \sqrt{4} = x + 2$ Explain to your friend why they have made an error.

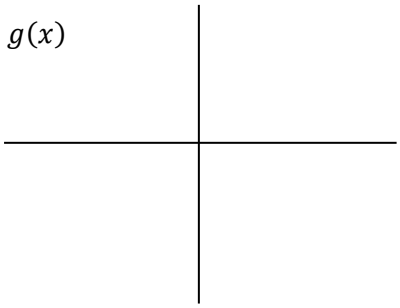
26) Your friend protests that the expressions are equal when they plug in zero for *x*.

Graph $f(x) = \sqrt{x^2+4}$ and $g(x) = x + 2$ Then refer to the graphs to explain to your friend why $f(0) = g(0)$ but in general $f(x)$ does not equal $g(x)$

$f(x)$



$g(x)$



27) Your friend is still struggling. Later on they see the following function and simplify it like so:

$f(x) \quad = (x+1)^2$

$= x^2 + 1^2$

$= x^2 + 1$

1) Plug in specific x values and compare them to $x^2 + 1$ to convince your friend that they cannot simplify this way.

2) Use a graph to convince your friend that simplifying this way is not valid.

28) Still later your friend sees you make the following simplification:

$f(x) = \sqrt{16x^2}$

$= 4x$ Your friend is upset, nay! Betrayed! Can you convince them that your simplification is allowed? Or have you made a mistake?

You will be tested on your knowledge of exponent rules.
www.mathsisfun.com/algebra/exponent-laws.html
For more practice questions, along with step-by-step answers, see here:
www.mesacc.edu/~scotz47781/mat120/notes/exponents/review/review_practice.html#

29) $(6x^3y)(-12xy)$

30) $(-3)^3$

31) $-7(a^5b^2)^3$

32) $(-1)^{73}$

33) $(-2a^5b^2)^3$

34) $10x^3y^2 - 6x^3y^2$

35) $(6^a)^4$

36) $(-2)^4$

37) $y^{5x} \cdot y^{4x}$

38) $14y(y^3) + 9y^2(y^2)$

39) $(6^a)(6^4)$

40) -5^2

41) $\left(\frac{1}{2}x^5\right)^4$

42) $\left(\frac{9}{21}abc\right)\left(\frac{7}{15}a^2bc^3\right)$

43) $(-cd)(-ce)(-de)$

44) $(2^6)(2^{m+5})(2^{4m})$

45) $\left(\frac{1}{4}x^2yz^5\right)(24xy)$

46) $4x(x^3) + 8x(x)$

47) $\frac{x^8}{x}$

48) $\frac{x^2y^4}{y^2z^2}$

49) $x^{-\frac{2}{3}} \cdot x^{\frac{4}{5}}$

50) $\frac{1}{x^{\frac{1}{2}}} \cdot x^{\frac{-1}{2}}$

Factor completely:

51) $8x^2 + 24x + 10$

52) $-12x^2 - 7x - 1$

53) $-3x^2 - 7x - 1$

54) $5x^2 + 25x + 20$

Multiply out

55) $(2x - 1)(x + 6)(x - 3)$

Simplify

58) $\frac{-x^2+6x-8}{-5x^2-5x+30}$

59) $\frac{2x^2+x-1}{x^2+3x-18} \div \frac{2x^2-5x+2}{x^2+4x-12}$

60) $\frac{2-5x}{x-10} + \frac{1}{3x+2}$

Describe what happens at $x = -2$ and $x = 7$. Note, different things happen at these two points.

61) $f(x) = \frac{x+2}{(x+2)(x-7)}$

Factor:

62) $x^2 + 15x + 36$

63) $x^2 - 25$

64) $6x^2 - 2x - 20$

65) $21x^2 + 8x - 4$

<u>Radicals</u>

Put the following in simple radical form if possible

66) $\sqrt[3]{2160x^3yz^0}$

67) $\left(\frac{x+3}{2x+7}\right)^{\frac{1}{2}}$

68) $\left(\frac{\sqrt[3]{4x+1}}{(2x)^2}\right)^{\frac{3}{2}}$

69) $3\sqrt{3} - \frac{6}{\sqrt{3}}$

70) $\frac{4\sqrt{7}+3\sqrt{2}}{5\sqrt{2}+2\sqrt{7}}$

Solve. Note any extraneous solutions. (Extraneous solutions are *x* values that result in division by zero.)

71) $\frac{2}{x}=\frac{5}{7}$

72) $\frac{1}{x+1}=\frac{7}{9}$

73) $x-2=\sqrt{x-2}+12$

74) $\frac{2}{\sqrt{x}+2}+\frac{\sqrt{x}}{\sqrt{x}-2}=\frac{x+4}{x-4}$

75) $\frac{4x+1}{x+1}=\frac{12}{x^2-1}+3$

76) $\frac{5x}{x-2}=7+\frac{10}{x-2}$

77) $\frac{2x-3}{x+3}=\frac{3x}{x+4}+1$

Answer in simplest radical form. Show all work. You can Google "simplest radical form" or read this page for more information https://www.themathpage.com/alg/simplify-radicals.htm

78) $\left(3\sqrt{7}\right)^2$

79) $\sqrt{72}$

80) $3x\sqrt{20}-\sqrt{5x}+5\sqrt{45x}$

81) $\sqrt{400000}$

82) $\sqrt[7]{-2187}+6\sqrt{196}$
(You can do this without a calculator.
Prepare to divide by 3 a few times.)

83) $\sqrt[4]{x^3b^{10}c^4}$

84) $\left(3\sqrt{6}\right)\left(4\sqrt{3}\right)$

85) $\left(3+\sqrt{6}\right)\left(4+\sqrt{3}\right)$

Simplify

86) $\frac{3}{2}+\frac{1}{4}$

87) $\frac{3x}{8}\cdot\frac{2}{9}$

88) $\frac{1}{3}\div\frac{4}{5}$

89) $\dfrac{\dfrac{2}{x+2}}{\dfrac{1}{x+2}+\dfrac{2}{x}}$

90) $\dfrac{\dfrac{15}{2x+2}}{\dfrac{6}{x}-\dfrac{1}{2}}$

91) $\dfrac{x^2-3x+2}{x^2+5x-6}$

92) $\dfrac{\dfrac{20}{x+1}}{\dfrac{1}{4}-\dfrac{7}{x+1}}$

93) $\dfrac{1}{\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}}$

94) $\dfrac{\dfrac{8}{2x+3}}{\dfrac{1}{2}-\dfrac{x}{2x+1}}$

Solve and write what is excluded from the domain. (Extraneous solutions that cause division by zero are excluded.)

95) $\dfrac{4x+1}{x+1}=\dfrac{12}{x^2-1}+3$

96) $\dfrac{6x}{x+4}+4=\dfrac{2x+2}{x-1}$

Graph and note values excluded from the domain and range. Show the points you plotted (in a table). Note the locations of asymptotes and indeterminate points.

97) $y=\dfrac{3x^2}{x^2-4}$

Miscellaneous:

98) Write this linear equation in slope-intercept form: $-7x+3y=15$

99) Write this linear equation in standard form: $y=\frac{3}{5}x+2$ (recall that standard form is $ax+by=c$)

100) $y=\pi x$ estimate the value of y when $x=3$

101) Find the inverse function of $f(x)=\frac{x+5}{x-2}$

102) Write an equation for the secant line to $f(x)=x^2+2$ on the interval $1\le x\le 3$

103) $\frac{2x}{y}+z^{-4}$

A) Write without fractions

B) Write without negative exponents

104) $\frac{2x}{4y^2}+\frac{6^0}{z^{-4}}$

A) Write without fractions

B) Write without negative exponents

105) Simplify $(x^{12}m^{25}a^{200}s^6)^2$

106) Simplify $(x^{12}+m^{25}a^0s^6)^2$

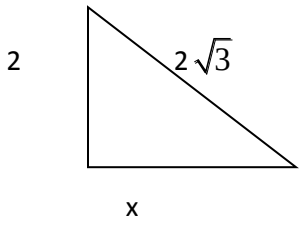
107) Draw a number line from $x=-5$ to $x=5$, then plot the next 6 values on that one number line:

- -5
- 5^{-1}
- -2^2
- $(-2)^2$
- -2^{-2}
- $(-2)^{-2}$

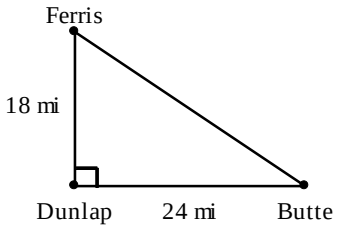
108) Which one of the following is not like the other?

- $11^{-3}(10+1)^3$
- $\frac{(-1)^3}{-1^{512}}$
- -997^0
- $\frac{(-1)^{-4}}{(1-0x)^2}$

109) Solve for x. Show all work and clear steps.



110) Leslie used the diagram to compute the distance from Ferris to Dunlap to Butte. How much shorter is the distance directly from Ferris to Butte than the distance Leslie found? Show all work and clear steps.



ANSWER KEY

(Note that answers were calculated by a human and therefore may contain errors.)

1)

	Coefficients	**Like Terms**	**Constants**	**Distinct Terms**
$3x+7$	3	None	7	2
$4m+(-3n)+n$	4, -3, 1	$-3n$ and n	None	2
$6kp+9k+kp-14$	6, 9, 1	$6kp$ and kp	-14	3
$-8y+6ab+7-3ba$	-8, 6, -3	$6ab$ and $-3ba$	7	3
$c+2c-5c+1$	1,2, -5	c and $2c$ and $-5c$	1	2
$5x^2-x^2y+4yx^2+xy^2$	5, -1, 4	$-x^2y$ and $4yx^2$	None	3
$\pi r^3+2r^2-2\pi+8r^3$	π, 2, 8	πr^3 and $8r^3$	-2π	3
$3\mathrm{x}\cdot\log(x)+x-\ln(7)\,x$	$3,-\ln(7)$	x and $-\ln(7)\,x$	None	2
$xe^x+2^x-5xe^x-8$	1,1,-5	xe^x and $-5xe^x$	-8	3

2)

	Coefficients	**Like Terms**	**Constants**	**Distinct Terms**
$5x+7y-7$	5,7	None	-7	3
$5a+2b-3a+4$	5,2,-3	$5a$ and $-3a$	4	3
$4st-5s+3ts+6$	4,-5,3	$4st$ and $3ts$	6	3
$7a^3b^4+b^4a^3+4a^3b-1$	7,1,4	$7a^3b^4$ and b^4a^3	-1	3
$\pi+x^2+\pi y^2-2\pi x^2$	π,1, π,-2π	x^2 and $-2\pi x^2$	π	3
$5\mathrm{x}^2\cdot\ln(x)+8x-\log(7)$	5, 8	None	$-\log(7)$	3
$3^x+x8^x-e^x+1$	1, 1, -1	None	1	4

3. Give an example of an expression:
Many possible answers. For instance,
Anything in the above tables.

4. Give an example of an equation:
$y = x$ or many other possible answers.
Must include an equals sign.

5. $8x$
The numerical coefficient is 8

6. Term: x
The value of the invisible coefficient is 1

7. A common mistake is to put this in your calculator as 8 / 2 π, but this is wrong. Instead you have to write 8 / (2π)

8. $x = 4$

9. You have to be able to factor 6 out of the entire numerator if you want to cancel.

10. The functions cross at $x = 0$ so they are equal there, but the functions are different everywhere else.

11. Answers will vary.

12. One explanation is that you can factor x from the denominator then cancel with the x in the numerator

13) $\frac{-17}{3}$

14) $\frac{13}{6}$

15) $-\frac{7}{36}$

16) $\frac{1}{12}$

17) $\frac{48}{11}$

18) $\frac{41}{90}$

19) $x = \frac{12}{7}$

20) $x = \frac{8}{9}$

21) $x = -4,\ 7$

22) $x = \frac{-1}{2},\ 1$

23) $x + 4$

24) $\frac{x+5}{x-1}$

25) You can't distribute exponents over addition or subtraction. $\sqrt{4+9} = \sqrt{13}$ it does not equal $2 + 3 = 5$

26) The functions cross at $x = 0$ so they are equal there, but the functions are different everywhere else.

27) Your friend should FOIL

28) The square root can be distributed across multiplication and division

29) $-72x^4y^2$

30) -27

31) $-7a^{15}b^6$

32) -1

33) $-8a^{15}b^6$

34) $4x^3y^2$

35) 6^{4a}

36) 16

37) y^{9x}

38) $23y^4$

39) 6^{a+4}

40) -25

41) $\frac{1}{32}x^{20}$

42) $\frac{1}{5}a^3b^2c^4$

43) $-c^2d^2e^2$

44) 2^{5m+9}

45) $6x^3y^2z^5$

46) $4x^4+8x^2$

47) x^7

48) $\frac{x^2y^2}{z^2}$ or $\left(\frac{xy}{z}\right)^2$

49) $x^{\frac{2}{15}}$

50) x^{-1}

51) $2(4x^2+6x+5)$

52) $-(3x+1)(4x+1)$

53) $-3(x-\frac{7+\sqrt{37}}{6})(x+\frac{-7+\sqrt{37}}{6})$

54) $5(x+4)(x+1)$

55) $2x^3+5x^2-39x+18$

58) $\frac{(x-4)}{5(x+3)}$

59) $\frac{x+1}{x-3}$

60) $\frac{-3(5x^2+x+2)}{(x-10)(3x+2)}$

61) There is a hole at $x=-2$ and an asymptote at $x=7$.

62) $(x+3)(x+12)$

63) $(x+5)(x-5)$

64) $2(3x+5)(x-2)$

65) $(7x-2)(3x+2)$

66) $6x\sqrt{10y}$

67) $\frac{\sqrt{(x+3)(2x+7)}}{2x+7}$

68) $\frac{\sqrt{4x+1}}{8x^3}$

69) $\sqrt{3}$

70) $\frac{7\sqrt{14}-13}{11}$ (uses diff of squares)

71) $x=\frac{14}{5}$

72) $x=\frac{2}{7}$

73) 18

74) $x=4$ (extraneous). No Solutions. (uses diff of squares)

75) $x=-2,\ 5$ (neither extraneous)

76) $x=2$ (extraneous). No Solutions.

76) $x=0,\frac{-11}{2}$ (Neither extraneous)

78) 63

79) $6\sqrt{2}$

80) $6x\sqrt{5}+14\sqrt{5x}$

81) $200\sqrt{10}$

82) 81

83) $c\sqrt[4]{x^3b^{10}}$ or $c\,x^{3/4}\,b^{5/2}$

84) $36\sqrt{2}$

85) $12+3\sqrt{3}+4\sqrt{6}+3\sqrt{2}$

86) $\frac{7}{4}$

87) $\frac{x}{12}$

88) $\frac{5}{12}$

89) $\frac{2x}{3x+4}$

90) $\frac{15x}{-x^2+11x+12}$

91) $\frac{x-2}{x+6}$

92) $\frac{80}{x-27}$

93) $\frac{abc}{bc+ac+ab}$

94) $\frac{32x+16}{2x+3}$

95) Excluded: $x = -1, 1$

96) Excluded: $x = -4, 1$

97) The graph has a downward parabola between two vertical asymptotes at x=2,-2. The other curves of the graph are above the horizontal asymptote at y=3. Domain: x not equal to 2,-2. Range: y <= 0 and y > 3.

98) $y = \frac{7}{3}x + 5$

99) $5y - 3x = 10$

100) $y \approx 9$

101) $f^{-1}(x) = \frac{2x+5}{x-1}$

102) $y = 4x - 1$

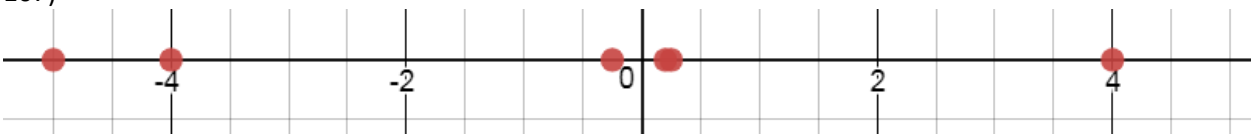
103) A) $2xy^{-1} + z^{-4}$
B) $\frac{2x}{y} + \frac{1}{z^4}$

104) A) $x2^{-1}y^{-2} + z^4$
B) $\frac{x}{2y^2} + z^4$

105) $x^{24}m^{50}a^{400}s^{12}$

106) $x^{24} + 2x^{12}m^{25}s^6 + m^{50}s^{12}$

107)



108) $-997^0 = -1$ All the others equal positive 1

109) Use Pythagorean Theorem. $x = \sqrt{8}$

110) $c = 30$ The distance is 12 miles shorter.